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Review: On Rank-One Perturbations of Normal Operators

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On rank-one perturbations of normal operators. (English summary)

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This article provides a strong result in support of the old conjecture [C. M. Pearcy, *Some recent developments in operator theory*, Amer. Math. Soc., Providence, R.I., 1978; MR0487495 (58 #7120)] that every rank-one perturbation (which is not a scalar multiple of the identity) of a diagonal normal operator has a nontrivial hyperinvariant subspace.

To be more specific, the authors prove that if D is a diagonalizable normal operator on a separable, infinite-dimensional, complex Hilbert space and $T = D + u \otimes v$ is a rank-one perturbation of D such that

(1) T is not a scalar multiple of the identity,
(2) the vectors u and v have Fourier coefficients $(\alpha_n)_{n=1}^{\infty}$ and $(\beta_n)_{n=1}^{\infty}$ with respect to an orthonormal basis that diagonalizes D that satisfy

$$\sum_{n=1}^{\infty}(|\alpha_n|^{2/3} + |\beta_n|^{2/3}) < \infty,$$

then T has a nontrivial hyperinvariant subspace.

The paper employs several results of [E.-J. Ionaşcu, Integral Equations Operator Theory **39** (2001), no. 4, 421–440; MR1829279 (2002c:47014)] and can be seen as a natural sequel to that article.

Reviewed by *Stephan R. Garcia*

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Note: This list reflects references listed in the original paper as accurately as possible with no attempt to correct errors.